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HYPERBOLIC AMPLIFIER
(HYDRODYNAMIC ATTENUATION EQUALIZER)

BY

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(Hydrodynamic Attenuation Equalizer)

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HYPERBOLIC AMPLIFIER

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A. NEED FOR THE HYPERBOLIC AMPLIFIER (Hydrodynamic Attenuation Equalizer)

The ocean wave signal supplied by the Mark IX Shore Wave Recorder System is subject to a selective attenuation. The error in the ocean wave signal caused by the selective attenuation is greatest for the waves of short wave lengths. This selective attenuation is caused by hydrodynamic attenuation of sub-surface pressure differentials by the body of water between the surface of the ocean and the Mark IX sub-surface, pressure-sensitive transducer. The Hyperbolic Amplifier (Hydrodynamic Attenuation Equalizer) discussed in this paper is the result of an attempt to find a device that could compensate for this hydrodynamic attenuation by introducing frequency selective amplification.

B. CHARACTERISTICS REQUIRED OF THE HYPERBOLIC AMPLIFIER (Hydrodynamic Attenuation Equalizer)

The selective attenuation of sub-surface pressure differentials of oscillatory waves by the layer of water between the ocean surface and the sub-surface, pressure-sensitive transducer has been shown to be expressed by the relationship

$$\frac{\text{Steady State}}{\text{Selective Attenuation}} = \frac{\cosh \frac{2\pi d}{L}}{\cosh \frac{2\pi d}{L} (1 - \frac{z}{d})} \quad (1)$$

where
 z = depth at which the pressure variation is being measured, in feet,
 d = depth of water at the transducer, in feet
 L = length of the surface wave, in feet

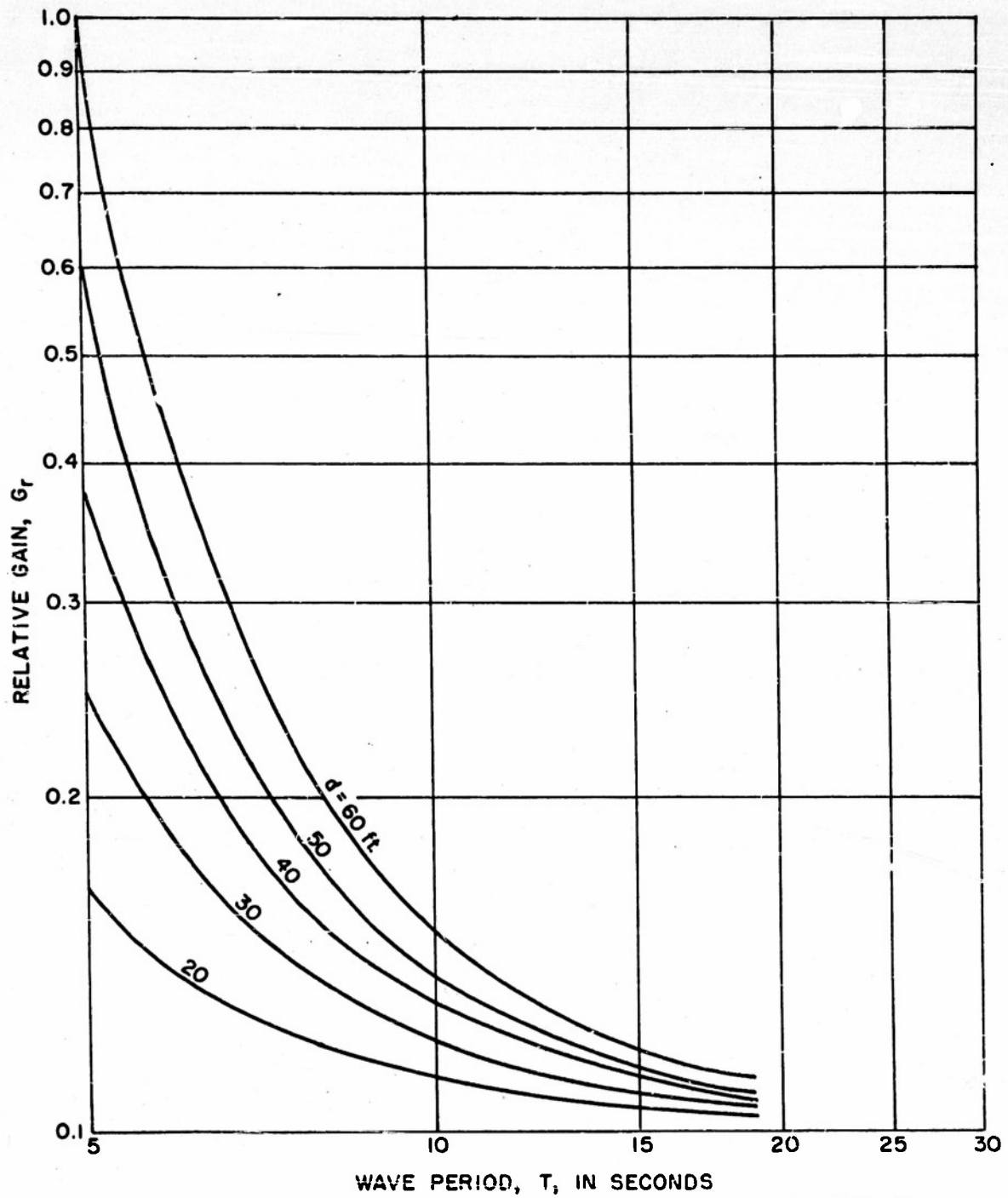
The recorded wave signal from the transducer does not enable a direct measurement of wave length, L ; it must be calculated from the wave period by the relationship

$$L = \frac{g}{2\pi} T^2 \tanh \frac{2\pi d}{L} \quad (2)$$

where
 g = gravitational constant
 T = wave period in seconds.

To compensate for this selective attenuation, the Equalizer must have a gain characteristic that is equal to the hydrodynamic attenuation characteristic. The required gain characteristics that the Equalizer must have for several different ocean depths are plotted in Figure 1.

2.



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RELATIVE GAIN VERSUS WAVE PERIOD
REQUIRED OF THE HYDRODYNAMIC ATTENUATION EQUALIZER
FOR VARIOUS DEPTHS OF OCEAN

FIGURE 1

C. CIRCUIT WITH ADEQUATE SELECTIVITY TO MEET THE REQUIREMENTS

If the Hydrodynamic Attenuation Equalizer were to be operated at audio or radio frequencies, electric circuits using resonant characteristics of inductive and capacitive elements could be used to give the necessary frequency selective amplification. However, at the low frequencies of ocean waves (0.04 to 0.2 cycles per second) the inductive reactance of the inductors is much less than their inherent resistance, causing the figure of merit (Q) of the resonant circuit to be so low that their frequency selectivity is inadequate for use as equalizers at ocean wave frequencies. The desired selectivity may be obtained, however, by using resistive and capacitive electric networks that rely on a phase shift principle for their selectivity. The parallel -T R-C network shown in Figure 2, is such a circuit and is used in this equalizer. Typical transmission characteristics of the parallel-T R-C network are shown in Figure 3.

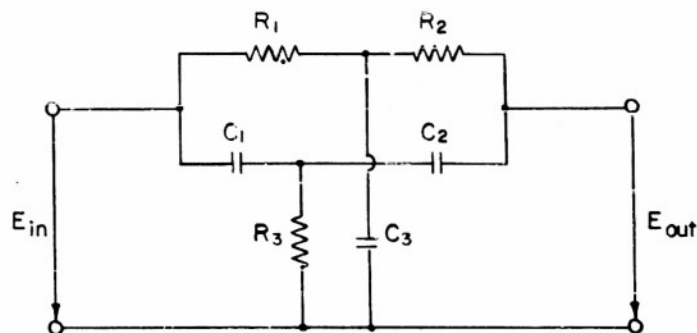


Fig. 2 - Parallel-T R-C network

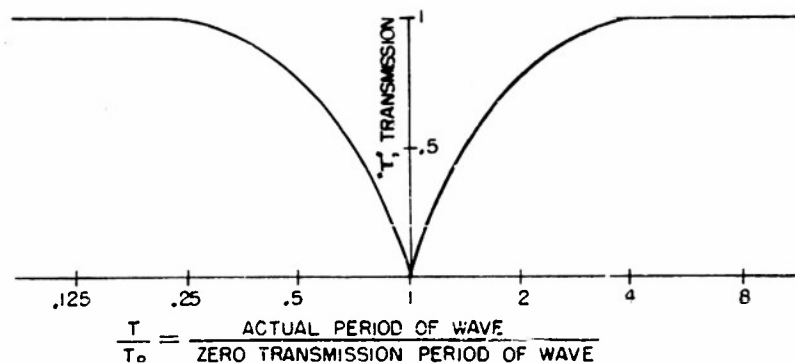


Fig. 3 - Typical transmission characteristics of unloaded parallel-T R-C network.

where transmission, T , is equal to the ratio of the voltage output to the voltage input of the parallel-T R-C network. When a load is connected to the output of the parallel-T, the selectivity is reduced. Much is written about the parallel-T R-C network in the literature*.

D. CIRCUIT GIVING THE RIGHT SHAPE GAIN CURVE

The selectivity of the parallel-T R-C network is adequate but its shape is inverse of the desired shape shown in Figure 1. The right shape of response curve for the Hydrodynamic Attenuation Equalizer can be obtained by using the parallel-T network in a negative feedback amplifier. The circuit of the negative feedback amplifier used is shown in Figure 4.

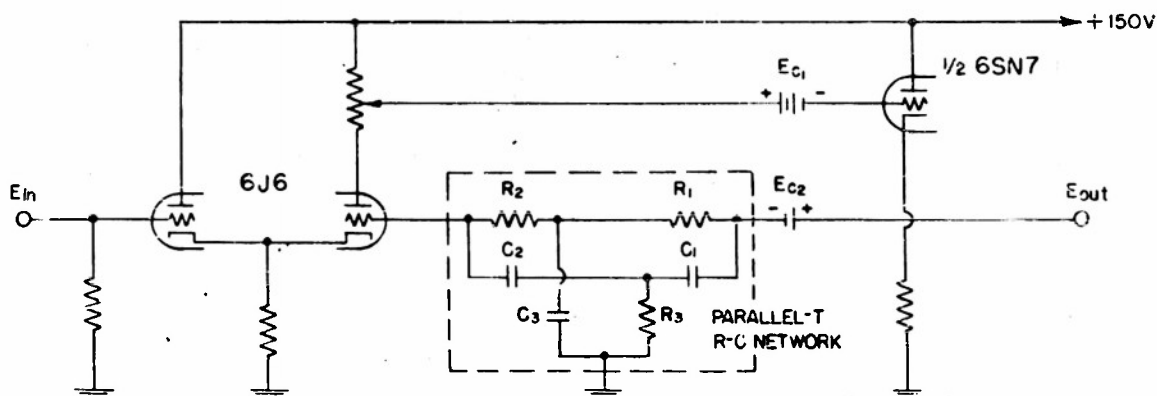


Fig. 4 - Negative feedback amplifier used as hydrodynamic attenuation equalizer

This particular form of a negative feedback amplifier was used to provide direct coupling from stage to stage so as not to introduce any frequency selectivity other than that of the parallel-T. It also lent itself nicely to the requirement that the load on the parallel-T R-C network must be a minimum for the parallel-T network to have maximum selectivity. To match the desired gain characteristics, nearly maximum selectivity of the parallel-T is required. The batteries are used to supply the grids of the tubes with the right operating potentials and still allow them to be direct coupled.

* L. Stanton, Theory and application of parallel-T R-C frequency selective networks, Proc. I.R.E., vol. 34, pp. 447-456, July 1946.

E.S. Furlington, Universal resistance-capacitance filter, U.S. Patent No. 2,354,141, July 1944.

W. N. Tuttle, Bridged-T and parallel-T null circuits for measurements at radio frequencies, Proc. I.R.E., Vol. 28, pp. 23-29, January 1940.

H. H. Scott, A new type of selective circuit and some applications, Proc. I.R.E., vol. 26, pp. 226-235, February 1938.

H. W. Augustadt, Electric filter, U.S. Patent No. 2,106,785; Feb. 1, 1938.

E. GAIN EXPRESSIONS OF HYDRODYNAMIC ATTENUATION EQUALIZER

The gain, G , of this electronic equalizer circuit can be expressed by the formula

$$G = \frac{A}{BT+1} \quad (3)$$

where A and B represent terms depending only on the circuit components other than the components of the parallel-T R-C network, and T is the transmission response of the parallel-T network. This expression is derived in Appendix A. The important expression for an equalizer, however, is the relative gain, G_r , which in this case is the absolute gain at the actual transmission, T , of the parallel-T divided by the absolute gain at the zero transmission $T = 0$.

$$G_r = \frac{G(T)}{G(T=0)} = \frac{\frac{A}{BT+1}}{\frac{A}{B \cdot 0 + 1}} = \frac{1}{BT+1} \quad (4)$$

Notice now that the relative gain has the right general shape to match the desired response curves shown in Figure 1.

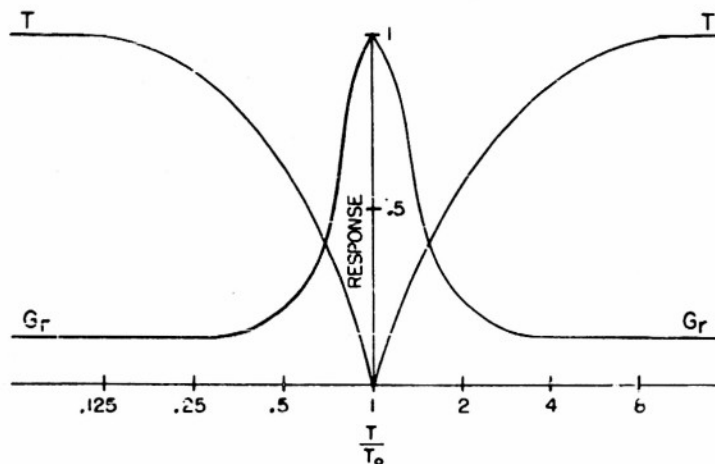


Fig. 5 - Typical graph of relative gain of equalizer and transmission versus wave period ratio

Note that if B is made equal to 9 by the right selection of components in the electronic equalizer other than those of the parallel-T, that the relative gain varies from 1 to 0.1 as T is varied from 0 to 1.

The right half of the gain curve of Figure 5 is used to match the desired response curves shown in Figure 1. By selecting the right value of B in equation 4, the end points of the equalizer's response curve are made to match the desired response. To obtain the closest match between the desired and actual response curves over the whole wave period range, it is necessary to select components of the parallel-T R-C network that will cause T , the transmission of the parallel-T network, to vary in a manner that will give the closest match.

F. CIRCUIT VALUES AND DESIGN PARAMETERS THAT GIVE THE CLOSEST MATCH BETWEEN THE DESIRED AND ACTUAL RESPONSE CURVES

The expression for the transmission, T , of a parallel-T R-C network, which is derived in the literature*, is

$$T = \frac{1}{1 + J \frac{T_0/T}{1 - (T_0/T)^2} U} \quad (5)$$

where U is the selectivity factor of the parallel-T network and can be expressed in terms of two convenient design parameters m and k , and J equals the square root of -1 .

$$U = \frac{1+m+2k}{k} \quad (6)$$

where $m = \frac{X_1}{R_1}$ and $k = \frac{R_2}{R_1}$ and where $X_1 = \frac{1}{\omega_0 C_1} = \frac{T_0}{2\pi C_1}$.

The transmission of the parallel-T R-C network is the most selective when U is the smallest. The smallest value of U occurs when 0 and $k = \infty$. Substituting these values into equation (6) and evaluating U we get:

$$U = \frac{1+0+2\infty}{\infty} = 2.$$

It is physically easy to obtain a value of $U = 2.2$ where it is impossible to obtain $U = 2$ and there is theoretically but a small difference between the resulting response when $U = 2$. When $U = 2.2$, the best match between the theoretically obtainable response curve of the electronic equalizer and the desired response curve is obtained when $B = 9.71$. A plot of this response for various values of zero transmission wave period, T , is given in Figure 6. Note that the same values of U and B give response curves that nearly match the desired response curves for different depths of ocean. All that has to be changed to match the desired response curve of a desired depth of ocean is the zero transmission period of the parallel-T R-C network. This is because for different depths of ocean the desired response only shifts its horizontal position and does not appreciably change its shape.

G. DESIGN EQUATIONS FOR THE PARALLEL-T R-C NETWORK

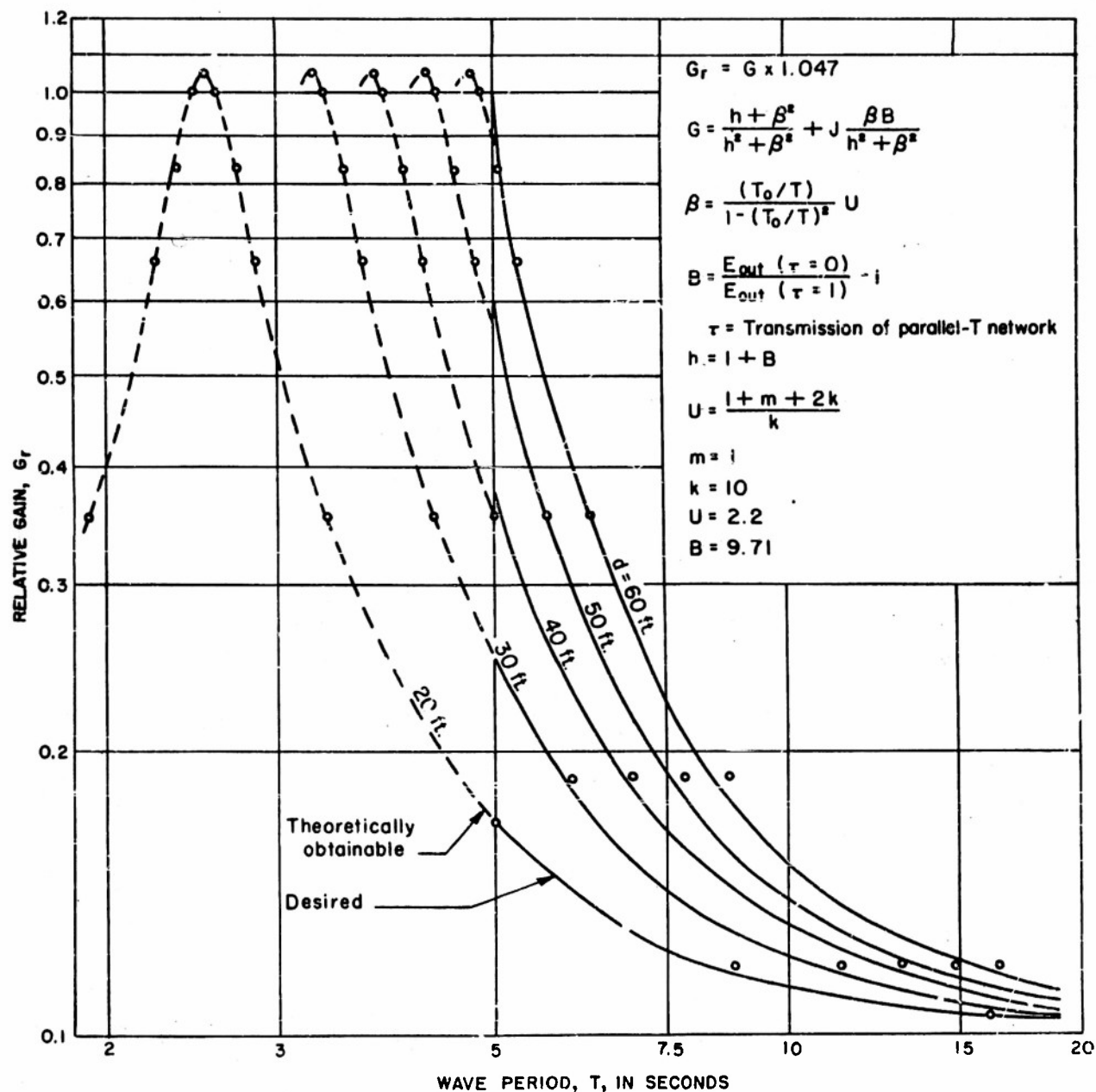
The manner in which the components of the parallel-T R-C network must be changed in order to change the zero transmission period without changing the selectivity factor U can be seen from the design equations of the parallel-T network

$$R_1 = \frac{T_0}{2\pi m C_1} \quad (7) \quad C_2 = \frac{m}{k} C_1 \quad (10)$$

$$R_2 = \frac{T_0 k}{2\pi m C_1} \quad (8) \quad C_3 = \frac{m}{k} (m+k) C_1 \quad (11)$$

$$R_3 = \frac{T_0 k}{2\pi (1+k) C_1} \quad (9) \quad U = \frac{m+2k}{k}$$

* Stanton, op. cit. p.4.



FAMILY OF THEORETICALLY OBTAINABLE GAIN CHARACTERISTICS
 OF HYDRODYNAMIC ATTENUATION EQUALIZER
 WHICH MOST NEARLY MATCHES FAMILY OF DESIRED GAIN CHARACTERISTICS
 FOR SEVERAL OCEAN DEPTHS

Equations 7, 8, 9, 10, and 11 are derived in Appendix B. U must be held constant, so design parameters m and k must be held constant. The above equations show that if m and k are held constant, only R_1 , R_2 , and R_3 need to be changed to change the zero transmission period of the parallel-T without changing the selectivity factor U . One component of the parallel-T network can be selected arbitrarily. Equations 7, 8, 9, 10, and 11 are set up so that C_1 may be selected arbitrarily and all the rest of the components are expressed in terms of C_1 , the zero transmission period and the design constants m and k . Selection of C_1 can best be done with an eye out for ease of obtaining the other components at whatever zero transmission period of the parallel-T that is required.

The value of T_0 , the zero transmission period of the parallel-T network, is fixed by the average depth of the ocean at the particular sub-surface transducer that the equalizer must work with.

Values of T for various depths of ocean up to 60 feet are obtained from Figure 6 and are tabulated in Table 1. Also in Table 1 there are tabulated values of R_1 , R_2 , and R_3 that are required in the parallel-T network when C_1 is equal to 10 microfarads. In Figure 7, the values in Table 1 are plotted.

Table 1.

Values of resistors and period of zero transmission, T_0 , of the parallel-T R-C network for several values of ocean depths, d , that give the gain characteristics shown in Figure 6 when C_1 of the parallel-T network is equal to 10 microfarads.

d Ft	T Sec	R_1 K Ω	R_2 K Ω	R_3 K Ω
60	4.73	75.4	754	68.5
50	4.28	68.1	681	62.0
40	3.79	60.4	604	54.85
30	3.26	51.95	519.5	47.2
20	2.53	40.3	403	36.62

H. METHOD OF OBTAINING THE DESIRED EQUALIZER CIRCUIT CONSTANTS OTHER THAN THE PARALLEL-T NETWORK CONSTANTS.

To match the desired gain curve the most nearly, the equalizer constant B should be adjusted to about 9.71. The value of A the absolute gain from input to output of the equalizer when $T = 0$ is not critical and can be seen in Appendix A to be of the order of B . Since the equalizer constant, B , is dependent on the gain characteristics of the vacuum tubes used, B will vary over a period of time and must be readjusted. The value of B can easily be controlled by the potentiometer in the plate circuit of the 6J6 (see Figure 4).

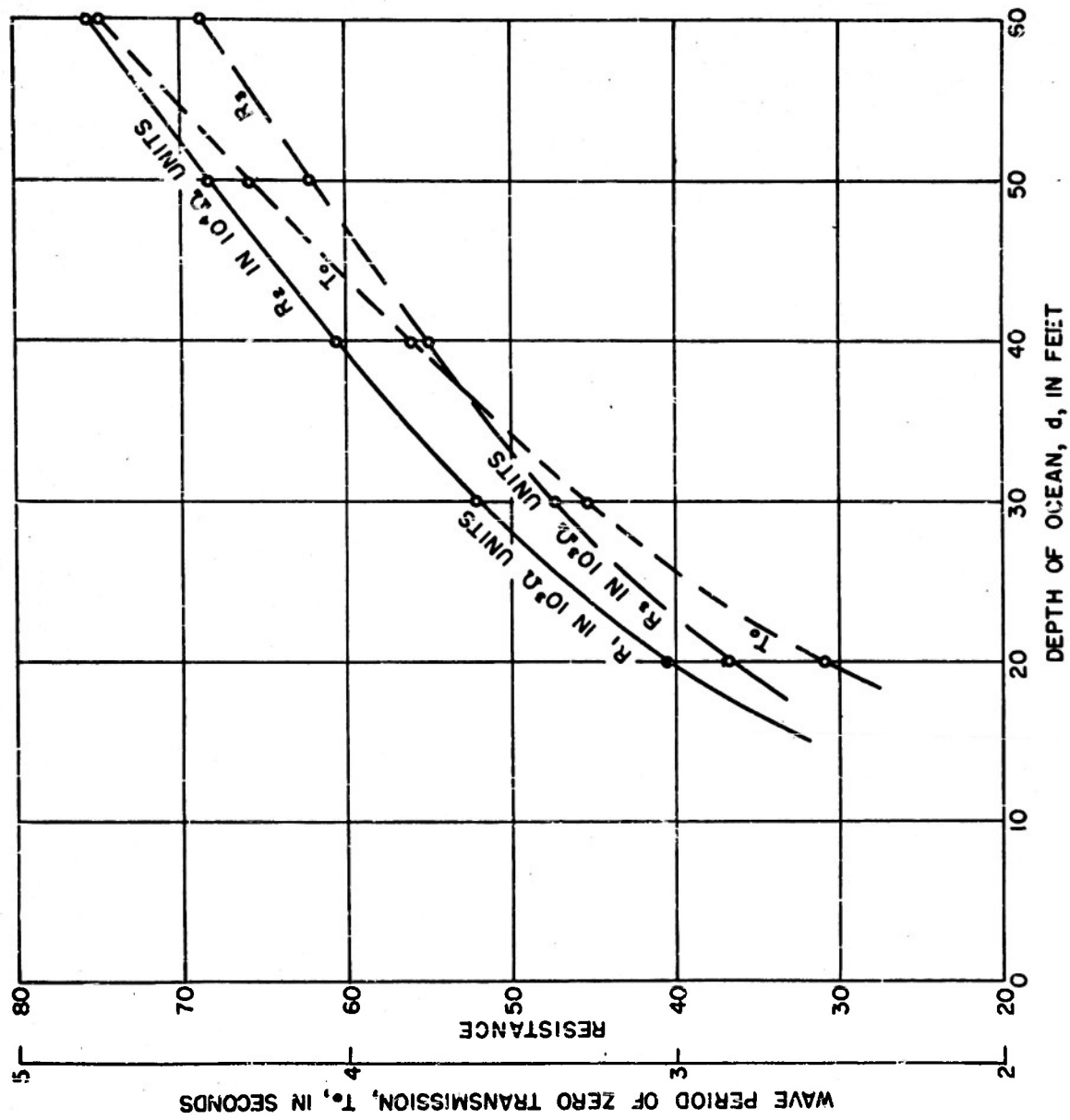
It is not necessary to depend on the accuracy of the circuit elements in setting B. Measurements of the output signal at an input signal have the same period as the zero transmission period of the parallel-T network and an input signal having a period that gives a transmission, T, of unity will enable B to be determined. This is because

$$\frac{E_{out}(T=1)}{E_{out}(T=0)} = \frac{\frac{E_{in}}{1+B+1}}{\frac{E_{in} A}{0+B+1}} = \frac{1}{B+1}$$

$$\therefore B = \frac{E_{out}(T=0)}{E_{out}(T=1)} - 1$$

I. EXPERIMENTAL DATA

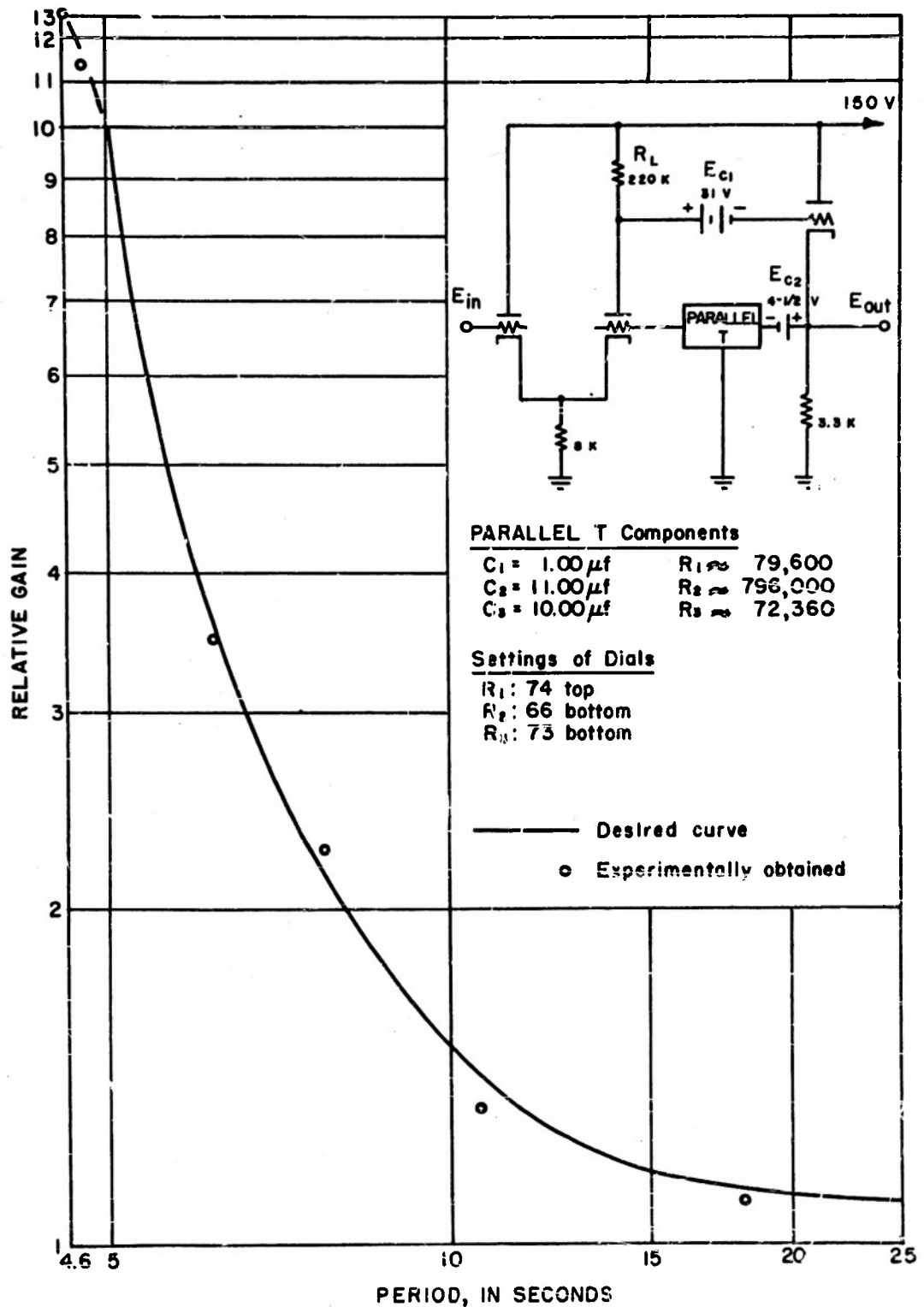
Experimental data was taken using a low frequency cam generator to supply the sinusoidal input of various wave periods, and a two channel Brush Recorder to determine the wave period and amplitude of the input and output of the equalizer. Figure 8 is an experimental plot of relative gain vs. wave period of the equalizer that very nearly matches the desired gain characteristics. Circuit components giving this response are indicated in Figure 8.



VALUES OF RESISTORS R_1 , R_2 & R_3 AND PERIOD OF ZERO TRANSMISSION OF PARALLEL-T R-C NETWORK VERSUS DEPTH OF OCEAN WHICH GIVE GAIN CHARACTERISTICS IN FIG.

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11.



RELATIVE GAIN OF HYDRODYNAMIC ATTENUATION EQUALIZER
 DETERMINED EXPERIMENTALLY

Derivation of Gain Formula of the Hydrodynamic Attenuation Equalizer.

First the equivalent circuit in terms of the small signal tube parameters and zero impedance voltage generators of the equalizer shown in Figure 9 is drawn in Figure 10.

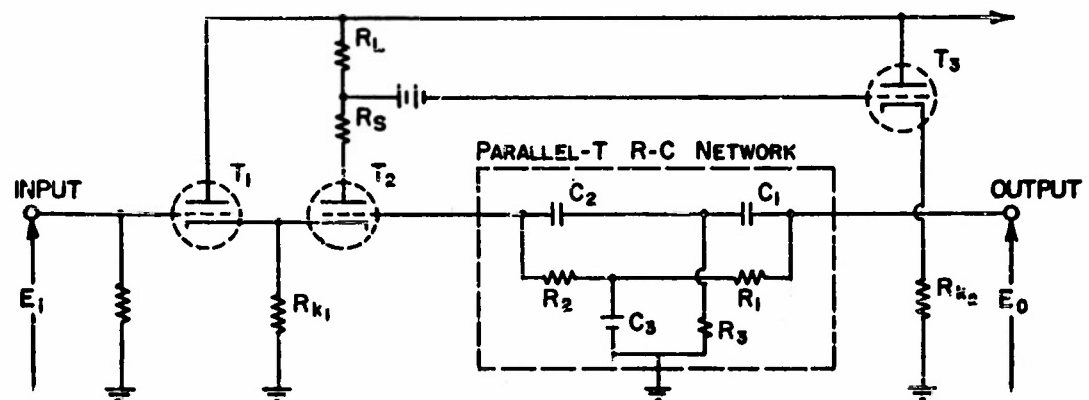


Fig. 9 - Hydrodynamic Attenuation Equalizer

The symbol T is used to represent the complex voltage transmission of the parallel-T network. The gain expression will now be found in terms of T and the circuit parameters other than those of the parallel-T network.

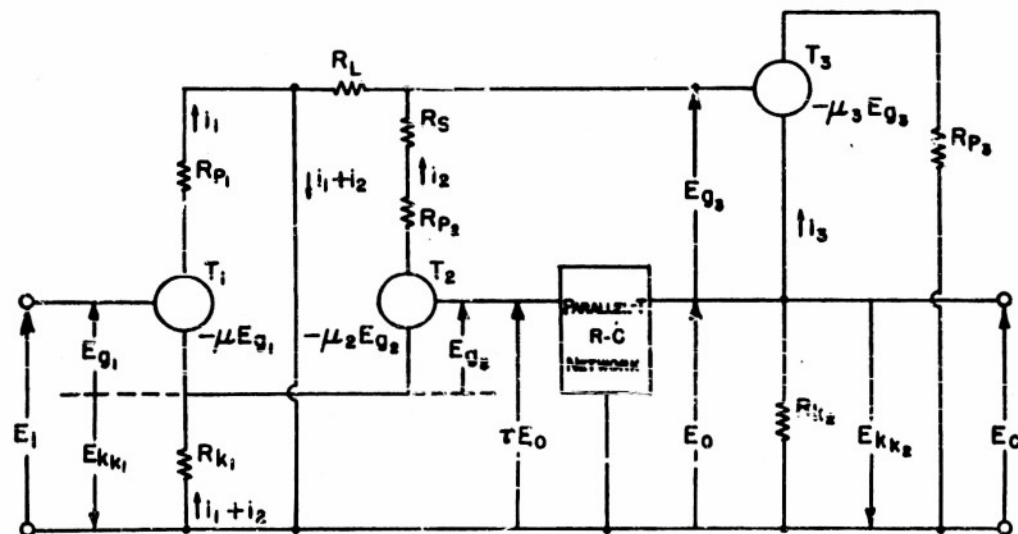


Fig. 10 - Equivalent Circuit of Hydrodynamic Attenuation Equalizer

$$E_{g_1} = E_1 + E_{R_{k_1}} \quad (1)$$

$$E_{R_{k_1}} = (i_1 + i_2) R_{k_1} \quad (2)$$

Substituting (2) into (1) gives

$$E_{g_1} = E_1 + (i_1 + i_2) R_{k_1} \quad (3)$$

$$i_1 (R_{P_1} + R_{k_1}) + i_2 R_{k_1} = -\mu_1 E_{g_1} \quad (4)$$

Substituting the expression for E_{g_1} in (3) into (4) gives, after simplifying,

$$i_1 [R_{P_1} + R_{k_1} (1 + \mu_1)] + i_2 R_{k_1} (1 + \mu_1) = -\mu_1 E_1 \quad (5)$$

Equation (5) is important and will be referred to later.

$$i_1 R_{k_1} + i_2 (R_{k_1} + R_L + R_S + R_{P_2}) = -\mu_2 E_{g_2} \quad (6)$$

Let $R_{k_1} + R_L + R_S + R_{P_2} = R_C$, signifying combined resistances, then

$$i_1 R_{k_1} + i_2 R_C = -\mu_2 E_{g_2} \quad (7)$$

$$\text{Since } E_{g_2} = \tau E_0 + (i_1 + i_2) R_{k_1} \quad (8)$$

and by substituting (8) into (7) we get

$$i_1 R_{k_1} (1 + \mu_2) + i_2 [R_C - R_{k_1} + R_{k_1} (1 + \mu_2)] = -\mu_2 \tau E_0 \quad (9)$$

$$\text{Since } E_0 = -E_{R_{k_2}} = -i_3 R_{k_2} \quad (10)$$

and by substituting (10) into (9) we get

$$i_1 R_{k_1} (1 + \mu_2) + i_2 [R_C - R_{k_1} + R_{k_1} (1 + \mu_2)] - i_3 \mu_2 \tau R_{k_2} = 0 \quad (11)$$

Equation (11) is important and will be referred to later.

$$i_3 (R_{k_2} + R_{P_3}) = -\mu_3 E_{g_3} \quad (12)$$

$$E_{g_3} = i_2 R_L + i_3 R_{k_2} \quad (13)$$

Substituting the expression for E_{g_3} in (13) for E_{g_3} in (12) gives

$$i_2 \mu_3 R_L + i_3 [R_{k_2} (1 + \mu_3) + R_{P_3}] = 0 \quad (14)$$

Equation (5), (11) and (14) give the three necessary expressions for solving for the three unknown currents i_1 , i_2 and i_3 . To find the gain, i_3 will be solved for from these three simultaneous linear equations by using a determinantal array. Bringing these three equations

together we have

$$i_1 [R_{P_1} + R_{k_1} (1 + \mu_1)] + i_2 R_{k_1} (1 + \mu_1) = -\mu_1 E_i \quad (5)$$

$$i_1 R_{k_1} (1 + \mu_2) + i_2 [R_C - R_{k_1} + R_{k_1} (1 + \mu_2)] - i_3 \mu_2 \tau R_{k_2} = 0 \quad (11)$$

$$i_2 \mu_3 R_L + i_3 [R_{k_2} (1 + \mu_3) + R_{P_3}] = 0 \quad (14)$$

$$i_3 = \frac{\begin{vmatrix} R_{P_1} + R_{k_1} (1 + \mu_1) & R_{k_1} (1 + \mu_1) & -\mu_1 E_i \\ R_{k_1} (1 + \mu_2) & R_C - R_{k_1} + R_{k_1} (1 + \mu_2) & 0 \\ 0 & \mu_3 R_L & 0 \end{vmatrix}}{\begin{vmatrix} R_{P_1} + R_{k_1} (1 + \mu_1) & R_{k_1} (1 + \mu_1) & 0 \\ R_{k_1} (1 + \mu_2) & R_C - R_{k_1} + R_{k_1} (1 + \mu_2) & -\mu_2 \tau R_{k_2} \\ 0 & \mu_3 R_L & R_{k_2} (1 + \mu_3) + R_{P_3} \end{vmatrix}} = N$$

$$N = -\mu_1 \mu_3 E_i R_L R_{k_1} (1 + \mu_2)$$

$$\begin{aligned} D &= [R_{P_1} + R_{k_1} (1 + \mu_1)] [R_C - R_{k_1} + R_{k_1} (1 + \mu_2)] [R_{k_2} (1 + \mu_3) + R_{P_3}] \\ &\quad + [R_{P_1} + R_{k_1} (1 + \mu_1)] [\mu_3 R_L] [\mu_2 R_{k_2}] \tau \\ &\quad - [R_{k_1} (1 + \mu_2)] [R_{k_1} (1 + \mu_1)] [R_{k_2} (1 + \mu_3) + R_{P_3}] \\ &= [R_{P_1} (R_C - R_{k_1}) + R_{k_1} (1 + \mu_1) (R_{P_1} + R_C - R_{k_1})] [R_{k_2} (1 + \mu_3) + R_{P_3}] \\ &\quad + [R_{P_1} + R_{k_1} (1 + \mu_1)] [\mu_2 \mu_3 R_L R_{k_2} \tau] \end{aligned}$$

$$\frac{E_o}{E_i} = \frac{-R_{k_2} i_3}{E_i} = \frac{-\mu_1 \mu_3 R_L R_{k_1} (1 + \mu_2) (-R_{k_2})}{R_P}$$

$$\text{Gain} = \underbrace{\left(\frac{\mu_1 \mu_3 R_L R_{k_1} (1 + \mu_2) R_{k_2}}{[R_{k_1} (R_C - R_{k_1}) (1 + \mu_1) + R_{P_1} (R_C + \mu_1 R_{k_1})] [R_{k_2} (1 + \mu_3) + R_{P_3}]} \right)}_A \underbrace{\left(\frac{1}{\frac{\mu_2 \mu_3 R_L R_{k_2} [R_{P_1} + R_{k_1} (1 + \mu_1)]}{R_{k_1} (R_C - R_{k_1}) (1 + \mu_1) + R_{P_1} (R_C + \mu_1 R_{k_1})}} \right)}_{B \tau + 1}$$

$$= A \frac{1}{B \tau + 1}$$

APPENDIX B

DERIVATION OF THE DESIGN EQUATIONS FOR THE VALUES OF THE COMPONENTS OF THE PARALLEL-T R-C NETWORK IN TERMS OF COMPONENT, C_1 , THE DESIGN PARAMETERS OF THE NETWORK, m & k , AND THE ZERO TRANSMISSION PERIOD, T_0

The derivation will be started from the two basic expressions for minimum transmission of parallel-T R-C networks that are found in the literature (see references at bottom of page 4). Equation 1 expresses the relationship that must be satisfied for the transmission of the parallel-T R-C network to be a minimum.

$$R_1 R_2 = (X_1 + X_2) X_3 \quad (1)$$

The symbols are the same as in Figure 2 on page 2, except that the X terms stand for the capacitive reactance of the capacitors having the corresponding subscript.

Equation 2 expresses the relationship that must be satisfied for the minimum transmission of the parallel-T R-C network to be zero.

$$X_1 X_2 = (R_1 + R_2) R_3 \quad (2)$$

If we let the ratios

$$X_1/R_1 = m \quad (3)$$

$$R_2/k_1 = k \quad (4)$$

where m and k are design parameters, we can find the values of the components of the parallel-T R-C network in terms of the zero transmission period desired, these design parameters which are determined by the selectivity factor U , requirements, and only one of the components. Note that the X 's in equations 1, 2 and 3 are the capacitive reactances of the capacitors at the zero transmission period.

First is obtained

$$X_2 = \frac{1+k}{m} R_3 \quad (5)$$

by rearranging equation 2 and substituting values for X_1 and R_2 from equations 3 and 4. Secondly we obtain

$$X_3 = \frac{R_1 2k}{R_1 + X_2} \quad (6)$$

by substituting values for X_1 and R_2 from equation 3 and equation 4 into equation 1.

Since we have six unknowns and only three requirements, three of the unknowns are arbitrary. Thus we will just arbitrarily let $X_2 = R_2$.

Then

$$X_2 = kR_1 \quad (\text{from equation 4}) \quad (7)$$

From this assumption we find further that

$$X_3 = \frac{R_1 2k}{R_1 m + k R_1} = \frac{k}{m+k} R_1 . \quad (8)$$

R_1 can be written in terms of T_0 by the relation of equation 3.

$$R_1 = \frac{X_1}{m} = \frac{T_0}{2\pi m C_1} . \quad (9)$$

We can now write

$$R_2 = \frac{k T_0}{2 \pi m C_1} \quad (10)$$

and

$$R_3 = \frac{m}{1+k} R_2 \quad (11)$$

or

$$R_3 = \frac{k T_0}{2 \pi (1+k) C_1} .$$

$$C_2 = \frac{T_0}{2 \pi X_2} \quad (12)$$

or

$$C_2 = \frac{T_0}{2 \pi R_2}$$

which is

$$C_2 = \frac{m}{k} C_1$$

$$C_3 = (m+k) \frac{m}{k} C_1 .$$

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